SOME NEW MULTIDIMENSIONAL COCHRAN–LEE
AND HARDY TYPE INEQUALITIES

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Abstract. A multidimensional Cochran-Lee operator is introduced and investigated in the frame of Hardy-type inequalities with parameters $0 < p \leqslant q < \infty$. Moreover, for the case $p = q$ and power weights even the sharp constant is derived, thus generalizing the original Cochran-Lee inequality to a multidimensional setting. As applications both several known but also new inequalities are pointed out.

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