NORM INEQUALITIES FOR PARALLEL SUMS OF OPERATORS

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Abstract. It is shown that if $A\!:\!B$ is the parallel sum of the positive definite operators A and B, then

$$\|A\!:\!B\| \leqslant \left\| \left(\frac{\|A\|\!:\!\|B\|}{\|A\|} \right)^2 A + \left(\frac{\|A\|\!:\!\|B\|}{\|B\|} \right)^2 B \right\|$$

$$\leqslant \frac{1}{2} \left(\|A\|\!:\!\|B\| + \frac{\|A\|\!:\!\|B\|}{\|A\|+\|B\|} \sqrt{ \left(\|A\|-\|B\|\right)^2 + 4\left\|\sqrt{A}\sqrt{B}\right\|^2 } \right).$$

These inequalities lead to a considerable improvement of the well-known inequality $\|A\!:\!B\| \leqslant \|A\|\!:\!\|B\|$ due to Anderson and Duffin $\left(\text{J. Math. Anal. Appl. } \mathbf{26} \text{ (1969), 576–594}\right)$. A lower bound for the norm of $A\!:\!B$ is also provided.

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