

## INEQUALITY OF HARDY-TYPE FOR $n$ -CONVEX FUNCTION VIA INTERPOLATION POLYNOMIAL AND GREEN FUNCTIONS

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**Abstract.** We obtain new results on the Hardy-type inequality in the general context, in terms of measure spaces with positive  $\sigma$ -finite measures. The connection is made between the difference operator derived from the Hardy-type inequality on the one hand and the expression containing the interpolating polynomial of Abel-Gontscharoff and the four Green functions on the other hand. We discuss the  $n$ -convexity of the function and consider the result depending on the parity of the indexes  $n$  and  $m$ . Further results are obtained by using the Hölder inequality for conjugate exponents  $p$  and  $q$ . Finally, we derive upper bounds for the remainder, obtained from the main result, using the Čebyšev functional. The Ostrowski-type bound for the generalized Hardy inequality is also given.

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