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WEIGHTED WEAK ESTIMATE FOR COMMUTATORS OF FRACTIONAL TYPE PARAMETRIC MARCINKIEWICZ INTEGRALS OVER NON–HOMOGENEOUS METRIC SPACES

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Abstract. Let $(\mathscr{X},d)$ be a metric space satisfying the geometrically doubling condition, and μ be a Borel measure satisfying the upper doubling condition. In this paper, the authors prove the weak type weighted $L^p(\omega)$ boundedness of the commutators $\mathscr{T}^b_{\beta,\rho,q}$ generated by the $RBMO(\mu)$ function b and the fractional type parametric Marcinkiewicz integral operator $\mathscr{T}_{\beta,\rho,q}$, which is defined over the non-homogeneous metric space $(\mathscr{X},d,\mu)$.

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