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THE SMALLEST EIGENVALUE OF LARGE HANKEL MATRICES ASSOCIATED WITH A SEMICLASSICAL LAGUERRE WEIGHT

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Abstract. The smallest eigenvalue of large Hankel matrices generated by a semiclassical Laguerre weight, $z^{\alpha}e^{-z^2+tz}$, where $z \in [0,\infty)$, $\alpha > -1$, and $t \in \mathbf{R}$, can be obtained through the asymptotics of the orthonormal polynomials $\mathscr{P}_n(z)$ with respect to this weight.

Mathematics subject classification (2020): 15B57, 26D20, 42C05, 65R10.
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