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ON THE JOINT NUMERICAL RADIUS OF GENERALIZED SPHERICAL ALUTHGE TRANSFORMS OF OPERATORS

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Abstract. In this paper, we generalize and refine several operator inequalities involving the joint numerical radius and the joint operator norm of spherical Aluthge transform to generalized spherical Aluthge transforms. Moreover, we investigate the link between nontrivial joint invariant subspaces of the generalized spherical Aluthge transform and the original commuting d-tuples of bounded operators.

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