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ON THE MINIMUM RANK OF DISTANCE MATRICES

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Abstract. Let $X = \{x_1,\dots,x_n\}$ be a finite set endowed with a metric d. The matrix $A = (d(x_i,x_j))_{n\times n}$ is called a distance matrix. In this paper we discuss about the minimum rank that can be achieved by an $n\times n$ distance matrix and prove that the rank of every 5×5 and 6×6 distance matrix is not less than 4.

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