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APPROXIMATION BY DE LA VALLÉE POUSSIN TYPE MARCINKIEWICZ MATRIX TRANSFORM MEANS OF WALSH–FOURIER SERIES

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Abstract. In the present paper, we discuss the rate of the approximation by the de la Vallée Poussin type Marcinkiewicz matrix transform of Walsh-Fourier series in $L^p(G^2)$ spaces ($1 \leqslant p < \infty$) and in $C(G^2)$. Namely, we prove

$$\left\|\sigma^T_{m,n}(f) - f\right\|_p \leqslant c \sum_{i=1}^{2} \omega_p^i \left(f, 2^{-|m|}\right)$$

in some special cases. Moreover, we give an application for functions in Lipschitz classes $\mathrm{Lip}(\alpha, p, G^2)$ ($\alpha > 0$, $1 \leqslant p < \infty$) and $\mathrm{Lip}(\alpha, C(G^2))$ ($\alpha > 0$).

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