SUBSPACE–HYPERCYCLIC CONDITIONAL WEIGHTED COMPOSITION OPERATORS ON L^p–SPACES

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Abstract. A conditional weighted composition operator $T_u : L^p(\Sigma) \to L^p(\mathscr{A})$ $(1 \leqslant p < \infty)$, is defined by $T_u(f) := E^{\mathscr{A}}(uf \circ \varphi)$, where $\varphi : X \to X$ is a measurable transformation, u is a weight function on X and $E^{\mathscr{A}}$ is the conditional expectation operator with respect to $\mathscr{A}$. In this paper, we study the subspace-hypercyclicity of T_u with respect to $L^p(\mathscr{A})$. First, we show that if φ is a periodic nonsingular transformation, then T_u is not $L^p(\mathscr{A})$-hypercyclic. The necessary conditions for the subspace-hypercyclicity of T_u are obtained when φ is non-singular and finitely non-mixing. For the sufficient conditions, the normality of φ is required. The subspace-weakly mixing and subspace-topologically mixing concepts are also studied for T_u. Finally, we give an example which is subspace-hypercyclic while is not hypercyclic.

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