ON (n, p)–TH VON NEUMANN–JORDAN
CONSTANTS FOR BANACH SPACES

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Abstract. In this paper, we introduce and discuss the upper, lower, upper modified and lower modified (n, p)-th von Neumann-Jordan constant for $p \geqslant 1$, which is a further generalization for von Neumann-Jordan constant in Banach spaces. We give some relationships between these constants and characterize uniformly non-l_n^1 Banach spaces by upper and upper modified (n, p)-th von Neumann-Jordan constant. Moreover, the exact value of this constant is calculated for the spaces l^p, L^p and $l_\infty - l_1$.

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