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HARDY–STEKLOV OPERATORS ON TOPOLOGICAL MEASURE SPACES

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Abstract. We give necessary and sufficient conditions on non-negative weights u,v and measures μ,ν in the inequality

$$\left(\int_\Omega |Tf(x)|^q u(x)d\mu(x)\right)^{1/q} \leqslant C\left(\int_\Omega |f(x)|^p v(x)d\nu(x)\right)^{1/p}.$$

Here the integral operator T is a Hardy-Steklov type operator associated with a family of open subsets $\Omega(t)$ of an open set Ω in a Hausdorff topological space X; μ,ν are σ-additive Borel measures, and $1<p<\infty$, $0<q<\infty$. The integration in T is over domains of type $\Omega(b(t))\setminus\Omega(a(t))$ where a,b are non-negative, increasing, continuous functions on $[0,\infty)$ that vanish at zero, tend to ∞ at ∞ and satisfy $a(t)<b(t)$ for $t\in(0,\infty)$. Previously such results have been known for an operator on a subset of a Euclidean space.
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