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PARTITIONS INTO m–TH LEHMER NUMBERS AND k–TH POWER RESIDUES IN $\mathbb{Z}_p$

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Abstract. Let p be a prime, $\mathbb{Z}_p^* = \{1,2,\ldots,p-1\}$, m,c be integers with $m \geqslant 2$, and $\mathscr{L}_m(c) = \{x | x \in \mathbb{Z}_p^*, 2 \nmid (x+(cx^m)_p)\}$, where $(cx^m)_p$ denotes the least positive residue modulo p. In this paper, we study the representation of any element of $\mathbb{Z}_p$ as sum of a m-th Lehmer number $l \in \mathscr{L}_m(c)$ and a k-th power residue in $\mathbb{Z}_p$, and give an inequality for the number of representations. Moreover, using the algorithm we provided, we examined all the cases for some pairs (k,m) by computer. We also analyzed the time complexity of the algorithm and illustrated that it is extremely difficult to verify all the cases up to the bound of p for larger km.

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