A NOTE ON ESTIMATES OF THE NEWTONIAN
POTENTIAL ON BOUNDED DOMAINS

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Abstract. In this paper, we investigate the regularity of the operator K on a smooth bounded domain in $\mathbb{R}^d$ given by convolution against the Newtonian potential. We show that the gain in L^p-Sobolev spaces agrees with elliptic regularity. We also establish L^p-Sobolev to L^q-Sobolev bounds as well as bounds from L^∞-Sobolev spaces to Hölder spaces.

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