Mathematical
Inequalities
& Applications
Volume 27, Number 3 (2024), 703–713
doi:10.7153/mia-2024-27-49

PARTITIONING BOUNDED SETS IN SYMMETRIC SPACES INTO SUBSETS WITH REDUCED DIAMETER

XINLING ZHANG* AND CHAN HE

Abstract. Borsuk's problem on partitioning bounded sets into sets having smaller diameters is considered. For each positive integer m and each n-dimensional Banach space X, let $\beta(X,m)$ be the infimum of $\delta \in (0,1]$ such that each bounded set $A \subseteq X$ with diameter 1 can be partitioned into m subsets whose diameters are at most δ. With the help of characterizations of complete sets in ℓ_1^3, we prove that $\beta(\ell_1^3,8) \leqslant 0.75$. By using the stability of $\beta(X,m)$ with respect to X in the sense of Banach-Mazur metric and estimations of the Banach-Mazur distance between ℓ_p^n and ℓ_q^n, we show that $\beta(\ell_p^3,8) \leqslant 0.88185$ holds for each $p \in [1,\infty]$. This improves a recent result of Y. Lian and S. Wu. Furthermore, we prove that $\beta(X,2^3) < 1$ when X is a three-dimensional Banach space symmetric with the natural basis $\{e_i \mid i \in [3]\}$ and satisfies $\alpha(X) = \left\| \sum_{i\in[3]} e_i \right\| > 9/4$.

Mathematics subject classification (2020): 46B20, 46B04.
Keywords and phrases: Banach-Mazur distance, Borsuk's partition problem, complete set, ℓ_p^n space.

REFERENCES

[1] V. BOLTYANSKI, H. MARTINI AND P. S. SOLTAN, *Excursions into Combinatorial Geometry*, Universitext. Springer, Berlin, 1997.
[2] A. BONDARENKO, *On Borsuk's conjecture for two-distance sets*, Discrete Comput. Geom. **51** (2014), 509–515.
[3] K. BORSUK, *Drei Sätze über die n-dimensionale euklidische Sphäre*, Fund. Math. **20** (1933), 177–190.
[4] H. G. EGGLESTON, *Covering a three-dimensional set with sets of smaller diameter*, J. London Math. Soc. **30** (1955), 11–24.
[5] T. JENRICH AND A. E. BROUWER, *A 64-dimensional counterexample to Borsuk's conjecture*, Electron. J. Combin. **21** (2014), Paper 4.29.
[6] J. KAHN AND G. KALAI, *A counterexample to Borsuk's conjecture*, Bull. Amer. Math. Soc. **29** (1993), 60–62.
[7] Y. LIAN AND S. WU, *Partition bounded sets into sets having smaller diameters*, Results Math. **76** (2021), Paper No. 116.
[8] H. MARTINI AND S. WU, *Complete sets need not be reduced in Minkowski spaces*, Beitr. Algebra Geom. **56** (2015), 533–539.
[9] J. P. MORENO AND R. SCHNEIDER, *Structure of the space of diametrically complete sets in a Minkowski space*, Discrete Comput. Geom. **48** (2012), 467–486.
[10] J. PERKAL, *Sur la subdivision des ensembles en parties de diamètre inférieur*, Colloq. Math. **1** (1947), 45.
[11] N. TOMCZAK-JAEGERMANN, *Banach-Mazur Distances and Finite-dimensional Operator Ideals*, Pitman Monographs and Surveys in Pure and Applied Mathematics, **38**, Longman Scientific & Technical, Harlow; copublished in the United States with John Wiley & Sons, Inc., New York, 1989.
[12] L. YU AND C. ZONG, *On the blocking number and the covering number of a convex body*, Adv. Geom. **9** (2009), 13–29.

Paper MIA-27-49

[13] L. ZHANG, L. MENG, AND S. WU, *Banach-Mazur distance from ℓ_p^3 to ℓ_∞^3*, Math. Notes. **114** (2023), 1045–1051.
[14] C. ZONG, *Borsuk's partition conjecture*, Jpn. J. Math. **16** (2021), 185–201.