

MONOTONICITY OF A TRACE RELATED TO TSALLIS RELATIVE OPERATOR ENTROPY

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Abstract. In this paper, for each $\alpha \in [0, 1]$ and two positive semidefinite matrices A and B , we show the monotonicity decreasing property on q of $-\text{Tr}[A^{1-q}T_{\frac{\alpha}{q}}(A^q||B^q)]$ for $0 < \alpha < q < 1$, which implies an Ando-Hiai result that complements Hiai-Petz inequality as $q \downarrow 0$, where $T_{\alpha}(A||B) = \frac{A^{\alpha}B-A}{\alpha}$.

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