

INEQUALITIES FOR FUSION FRAMES IN HILBERT SPACES WITH NEW STRUCTURES

ZHONG-QI XIANG*, CHUN-XIA LIN AND XIANG-CHUN XIAO

Abstract. This paper is concerned with inequalities for fusion frames in Hilbert spaces. With the help of two inequalities on bounded linear operators, several bilateral inequalities associated with a parameter for fusion frames are established, which, comparing to existing ones on this topic, possess new structures.

Mathematics subject classification (2020): 42C15, 42C40, 46C50, 47B40.

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