SINGULAR VALUES OF COMPACT OPERATORS VIA OPERATOR MATRICES

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Abstract. This paper finds new upper bounds for the singular values of certain operator forms. Compared with the existing literature, numerous numerical examples will be given to show that the obtained forms add a new set of independent bounds, that are incomparable with some celebrated known results.

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