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MIXED TYPE WEIGHTED INTEGRAL INEQUALITIES FOR THE HARDY–STEKLOV INTEGRAL OPERATORS

RAJIB HALOI AND DURANTA CHUTIA*

Abstract. We characterize the weights ω, ρ, ϕ and ψ for which the integral operator of Hardy-Steklov type, $\mathscr{I}f(t) = h(t)\int_{\alpha(t)}^{\beta(t)} K(t,z)f(z)w(z)dz$ satisfies weak type mixed modular inequalities of the form

$$\mathscr{U}^{-1}\Big(\int_{\{\mathscr{I}f>\gamma\}} \mathscr{U}(\gamma\omega)\rho\Big) \leqslant \mathscr{V}^{-1}\Big(\int \mathscr{V}(Cf\phi)\psi\Big),$$

where the functions α and β are increasing and the kernel K satisfies certain monotone conditions. We also prove the following mixed integral inequalities of the extra-weak type under appropriate conditions on the weights ω, ϕ and ψ.

$$\omega\Big(\{\mathscr{I}f>\gamma\}\Big) \leqslant \mathscr{U}\circ\mathscr{V}^{-1}\Big(\int \mathscr{V}\Big(\frac{Cf\phi}{\gamma}\Big)\psi\Big).$$

Further, we discuss the above two integral inequalities for the adjoint of the integral operator of Hardy-Steklov type.

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