

WEIGHTED NORM INEQUALITIES FOR SCHRÖDINGER OPERATORS ON VARIABLE LEBESGUE SPACES

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Abstract. In this work we show that many operators from harmonic analysis associated with the semigroup generated by the Schrödinger operator $\mathcal{L} = -\Delta + V$ in \mathbb{R}^n , where $n > 2$ and the non-negative potential V belongs to the reverse Hölder class RH_q with $q > n/2$ – such as maximal operators, the Littlewood–Paley function, pseudo-differential operators, singular integrals, and their commutators – are bounded on the weighted variable Lebesgue space $L^{p(\cdot)}(w)$. We do so by applying the theory of weighted norm inequalities and extrapolation.

Mathematics subject classification (2020): Primary 42B25; Secondary 35J10.

Keywords and phrases: Norm inequalities, Schrödinger operator, variable Lebesgue spaces, extrapolation, weights.

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