A WEIGHTED GENERALISATION OF CARLEMAN'S INEQUALITY

SERGI ARIAS* AND SALVADOR RODRÍGUEZ-LÓPEZ

Abstract. In this paper, we present a generalisation of the classical inequality of Carleman, which we obtain by an elementary argument based on log-convexity and Hölder's inequality. As a consequence, we recover some other classical estimates such as the Pólya-Knopp inequality.

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