OPTIMAL ESTIMATES IN MUSIELAK–ORLICZ SPACES FOR A PARABOLIC SCHRÖDINGER EQUATION

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Abstract. Let $d \geqslant 2$ and u be a strong solution to the parabolic Schrödinger equation

$$u_t - \Delta u + Vu = f \qquad \text{in } \mathbb{R}_T^d := \mathbb{R}^d \times (0, T).$$

We show that

$$\|u_t\|_{L^{\varphi(\cdot)}(\mathbb{R}_T^d)} + \|D^2 u\|_{L^{\varphi(\cdot)}(\mathbb{R}_T^d)} + \|Vu\|_{L^{\varphi(\cdot)}(\mathbb{R}_T^d)} \leqslant C \|f\|_{L^{\varphi(\cdot)}(\mathbb{R}_T^d)}$$

under suitable conditions on the Musielak-Orlicz function φ and the potential V.

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