

## PROOFS FOR ALZER'S CONJECTURE ON THE GENERALIZED LOGARITHMIC MEAN

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*Abstract.* One defines with K. B. Stolarsky in 1975 the generalized logarithmic mean of two positive numbers. In 1986, H. Alzer posed a conjecture on the generalized logarithmic mean, obtaining some partial results. Using properties of hyperbolic functions, two inequalities for the generalized inverse harmonic means are established in this paper. Based on these inequalities, a proof of Alzer's conjecture is given.

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