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IMPROVED L^p–L^q HARDY INEQUALITIES

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Abstract. In this note, we obtain a new version of the Hardy inequality which covers the recent inequality of Frank, Laptev, and Weidl derived in [2] and improves the result of Persson and Samko established in [8]. It gives new results in one dimension. We analyse radial and non-radial multidimensional versions of the considered inequality as consequences.

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