

SOME INEQUALITIES AND EQUATIONS OF g -FRAME OPERATOR MULTIPLIERS FOR FINITE GROUP REPRESENTATIONS

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Abstract. In wavelet theory, discussing the characterizing of wandering vector multipliers or frame vector multipliers for unitary representations of various groups is a very interesting problem. However, even in the case of Abelian groups, the characteristics of frame vector multipliers are still unknown. The purpose of this paper is to study g -frame operator multipliers by combining the unitary representation of finite groups with operator theory. Firstly, some new inequalities and equations that reflect the properties and characterizations of g -frame operator multipliers are discussed. With the help of group representation theory and operator theory, some necessary conditions such that a unitary operator is a g -frame operator multiplier can be found. Next, the g -frame operator multiplier for the more general case is discussed. In particular, the relationship between the g -frame operator multiplier of the direct sums of irreducible subrepresentations and the g -frame operator multiplier of the subrepresentations is obtained.

Mathematics subject classification (2020): 42C15, 46C07.

Keywords and phrases: g -frames, frames, group representations, g -frame operator multipliers.

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