ON THE σ–BALANCING PROPERTY OF MULTIVARIATE GENERALIZED QUASI–ARITHMETIC MEANS

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Abstract. The aim of this paper is to characterize the so-called σ-balancing property in the class of generalized quasi-arithmetic means. In general, the question is whether those elements of a given family of means that possess this property are quasi-arithmetic.

The first result in the latter direction is due to G. Aumann who showed that a balanced complex mean is necessariliy quasi-arithmetic provided that it is analytic. Then Aumann characterized quasi-arithmetic means among Cauchy means in terms of the balancing property. These results date back to the 1930s. In 2015, Lucio R. Berrone, generalizing balancedness, concluded that a mean having that more general property is quasi-arithmetic if it is symmetric, strict and continuously differentiable. A common feature of these results is that they assume a certain order of differentiability of the mean whether or not it is a natural condition.

In 2021, the balancing property was characterized in the family of generalized quasi-arithmetic means of two variables under only natural conditions, namely continuity and strict monotonicity of their generating functions. Here we extend the corresponding result for multivariate generalized quasi-arithmetic means by relaxing the conditions on the generating functions and considering the more general σ-balancing property.

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