

NECESSARY AND SUFFICIENT CONDITIONS FOR A MIXED BIVARIATE MEAN OF THREE PARAMETERS TO BE SCHUR m -POWER CONVEX

TAO ZHANG, BO-YAN XI AND FENG QI*

Abstract. In the work, by virtue of some techniques in the theory of majorization, the authors find necessary and sufficient conditions for a mixed bivariate mean of three parameters to be Schur m -power convex.

Mathematics subject classification (2020): Primary 26E60; Secondary 26A51, 26D20.

Keywords and phrases: Schur-convexity, Schur m -power convexity, necessary and sufficient condition, mixed mean.

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