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# APPLICATIONS OF UNITARILY DIAGONALIZABLE MATRICES IN AN INDEFINITE INNER PRODUCT SPACE TO MATRIX PARTIAL ORDERS

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*Abstract.* Necessary and sufficient conditions for the unitary diagonalization of normal matrices in an indefinite inner product space are given. As an application of unitary diagonalization, several new characterizations of the star partial order with respect to an indefinite inner product are established. The concepts of diamond order, space pre-order, and plus order are studied in the indefinite setting. Some relations among these matrix partial orders are proved.

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Paper MIA-28-01

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