

NEWTON-LIKE INEQUALITIES FOR LINEAR COMBINATIONS

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Abstract. We provide conditions under which linear combinations of normalized elementary symmetric polynomials satisfy some Newton-like inequalities. Namely, the log-concavity of the coefficients and nonnegativity of the arguments. We prove that such conditions are essential. That is, dropping any of these conditions leads to counterexamples. This settled a conjecture of Ren [C. Ren, *A generalization of Newton-Maclaurin's inequalities*, Int. Math. Res. Not. IMRN **5**, (2024), 3799–3822].

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