COVERING THE UNIT BALL OF ℓ_p^n WITH SMALLER BALLS AND RELATED INEQUALITIES

FEIFEI CHEN, SHENGHUA GAO, XIA LI AND SENLIN WU*

Abstract. Let B_p^n ($p \geqslant 1$) be the unit ball of ℓ_p^n and $\Gamma_m(B_p^n)$ be the smallest positive number γ such that B_p^n can be covered by m translates of γB_p^n. By using different configurations of translates of γB_p^n, we obtain a universal upper bound of $\Gamma_{2^n}(B_p^n)$ for fixed $p \in [1, \infty]$, a nontrivial upper bound for $\Gamma_{2^n}(B_p^n)$ for all $p \in [1, \infty]$ when n is small, and a useful upper bound of $\Gamma_{2^n}(B_p^n)$ when n and p are both large. It is still not clear whether there exists a constant $c \in (0, 1)$ such that $\Gamma_{2^n}(B_p^n) \leqslant c$ holds whenever $p \geqslant 1$ and $n \geqslant 2$.

Mathematics subject classification (2020): 46B20, 52A20, 52C17, 52A15.
Keywords and phrases: Convex body, covering functionals, entropy number.

REFERENCES

[1] U. BETKE AND M. HENK, *Intrinsic volumes and lattice points of crosspolytopes*, Monatsh. Math. **115** (1993), no. 1–2, 27–33.

[2] K. BEZDEK AND M. A. KHAN, *The geometry of homothetic covering and illumination*, Discrete Geometry and Symmetry, Springer Proc. Math. Stat., vol. 234, Springer, Cham, 2018, pp. 1–30.

[3] V. BOLTYANSKI AND I. Z. GOHBERG, *Stories about covering and illuminating of convex bodies*, Nieuw Arch. Wisk. (4) **13** (1995), no. 1, 1–26.

[4] V. BOLTYANSKI, H. MARTINI, AND P. S. SOLTAN, *Excursions into Combinatorial Geometry*, Universitext, Springer-Verlag, Berlin, 1997.

[5] L. FEJES TÓTH, G. FEJES TÓTH, AND W. KUPERBERG, *Lagerungen – arrangements in the plane, on the sphere, and in space*, Grundlehren der mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 360, Springer, Cham, [2023] ©2023, translated from the German [0057566], with a foreword by Thomas Hales.

[6] CHAN HE, H. MARTINI, AND SENLIN WU, *On covering functionals of convex bodies*, J. Math. Anal. Appl. **437** (2016), no. 2, 1236–1256.

[7] M. KOSSACZKÁ AND J. VYBÍRAL, *Entropy numbers of finite-dimensional embeddings*, Expo. Math. **38** (2020), no. 3, 319–336.

[8] M. LASSAK, *Covering a plane convex body by four homothetical copies with the smallest positive ratio*, Geom. Dedicata **21** (1986), no. 2, 157–167.

[9] XIA LI, LINGXU MENG, AND SENLIN WU, *Covering functionals of convex polytopes with few vertices*, Arch. Math. (Basel) **119** (2022), no. 2, 135–146.

[10] H. MARTINI, K. J. SWANEPOEL, AND G. WEISS, *The geometry of Minkowski spaces – a survey. I*, Expo. Math. **19** (2001), no. 2, 97–142.

[11] H. MARTINI AND SENLIN WU, *Concurrent and parallel chords of spheres in normed linear spaces*, Studia Sci. Math. Hungar. **47** (2010), no. 4, 505–512.

[12] G. PÓLYA AND G. SZEGŐ, *Problems and theorems in analysis. I*, Classics in Mathematics, Springer-Verlag, Berlin, 1998, Series, integral calculus, theory of functions, Translated from the German by Dorothee Aeppli, Reprint of the 1978 English translation.

[13] H. ROBBINS, *A remark on Stirling's formula*, Amer. Math. Monthly **62** (1955), 26–29.

[14] V. TEMLYAKOV, *A remark on entropy numbers*, Studia Math. **263** (2022), no. 2, 199–208.

[15] FEI XUE, YANLU LIAN, AND YUQIN ZHANG, *On Hadwiger's covering functional for the simplex and the cross-polytope*, 2021.

[16] CHUANMING ZONG, *A quantitative program for Hadwiger's covering conjecture*, Sci. China Math. **53** (2010), no. 9, 2551–2560.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com