

## NEW QUANTUM DIVERGENCES GENERATED BY MONOTONICITY INEQUALITY

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*Abstract.* In this paper, we introduce a new class of quantum divergences generated by monotonicity inequality. We also consider some related inverse problems for matrix means. As a consequence, we obtain some new characterizations of the trace property via monotonicity inequality.

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