

## APPROXIMATION IN GENERALIZED MORREY SPACES USING THE SECOND-ORDER DIFFERENCE

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*Abstract.* The issue of the denseness property of smooth functions has been considered in many studies. To allow approximation by smooth functions in generalized Morrey spaces, generalized Zorko spaces and vanishing-type spaces are defined. While the generalized Zorko space employs the first-order difference, we construct a subspace of the generalized Morrey space utilizing the second-order difference. We investigate its properties concerning approximation by smooth functions and its relation to the generalized Zorko space.

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