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ONE–SIDED MAXIMAL OPERATORS ON HERZ SPACES WITH VARIABLE EXPONENTS

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Abstract. This paper extends the boundedness of the one-sided maximal operators from the Lebesgue spaces with variable exponents to the one-sided Herz spaces with variable exponents. The main result generalizes the boundedness of the one-sided maximal operators on the classical Herz spaces.

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