Mathematical
Inequalities
& Applications
Volume 28, Number 2 (2025), 239–255
doi:10.7153/mia-2025-28-16

HARDY–HILBERT TYPE INEQUALITIES ON HOMOGENEOUS GROUPS–AN INTRODUCTION AND GENERALIZATION TO THE KERNEL CASE

MARKOS FISSEHA YIMER, LARS-ERIK PERSSON*, MICHAEL RUZHANSKY, NATASHA SAMKO AND TSEGAYE GEDIF AYELE

Abstract. There is a lot of information available concerning Hardy-Hilbert type inequalities in one or more dimensions. In this paper we introduce the development of such inequalities on homogeneous groups. Moreover, we point out a unification of several of the Hardy-Hilbert type inequalities in the classical case to a general kernel case. Finally, we generalize these results to the homogeneous group case.

Mathematics subject classification (2020): 26D10, 26D20, 12E30.
Keywords and phrases: Inequalities, Hardy's inequality, Hardy-Hilberts inequality, hyperbolic spaces, homogeneous groups, Cartan-Hadamard manifolds.

REFERENCES

[1] V. FISCHER AND M. RUZHANSKY, *Quantization on Nilpotent Lie Groups*, Progress in Mathematics, vol. 314. Basel, Switzerland: Birkhäuser, open access, 2016.
[2] G. B. FOLLAND AND E. M. STEIN, *Hardy Spaces on Homogeneous Groups*, Princeton University Press, Math. Notes, 28. Princeton University Press, NJ; University of Tokyo Press, Tokyo, 1982.
[3] G. H. HARDY, *Notes on some points in the integral calculus, LX*, Messenger of Math. **54** (1925), 150–156.
[4] G. H. HARDY, *Note on a theorem of Hilbert concerning series of positive terms*, Proc. of the London Math. Soc. **23** (2), (1925), 45–46.
[5] G. H. HARDY, J. E. LITTLEWOOD AND G. POLYA, *Inequalities*, Cambridge Univ. Press, Cambridge 1952.
[6] D. HILBERT, *Grundzuge einer allgemeinen theorie der linear integrolgleichungen*, Göttingen Nachr. (1906), 157–227.
[7] N. KARAPETIANTS AND S. SAMKO, *Equations with Involutive Operators*, Birkhäuser, Boston, 2001.
[8] J. KUANG, *On new extensions of Hilbert's integral inequality*, J. Math. Anal. Appl. **235** (2), (1999), 608–614.
[9] A. KUFNER, L.-E. PERSSON AND N. SAMKO, *Weighted inequalities of Hardy Type*, second edition, World Scientific Publishing Co. New Jersey, 2017.
[10] D. LUKKASSEN, L.-E. PERSSON AND S. SAMKO, *Some sharp inequalities for integral operators with homogeneous kernels*, J. Integral Appl. 2016, 2016:114, 18 pp.
[11] B. G. PACHPATTE, *A note on Hilbert type inequality*, Tamkang J. Math. **29** (4), (1998), 293–298.
[12] L.-E. PERSSON, *Lecture Notes*, College de France, Pierre-Louis Lions Seminar, Paris, France, November 2015 (48 pages).
[13] L.-E. PERSSON AND N. SAMKO, *On Hardy-type inequalities as an intellectual adventure for 100 years*, J. Math. Sci. (2024), 22 pp.
[14] M. RUZHANSKY AND D. SURAGAN, *Hardy Inequalities on Homogeneous Groups*, 100 years of Hardy Inequalities, Progr. Math. **327**, Birkhäuser/Springer, Cham, 2019.
[15] I. SCHUR, *Bemerkungen zur theorie der beschränkten bilinearformen mit unendlich vielen veränderlichen*, J. Reine Angew. Math. **140**, (1911), 1–28.

Paper MIA-28-16

[16] J. WEIJIAN AND G. MINGZHE, *An extended Hardy-Hilbert inequality and its applications*, J. Inequal. Pure Appl. Math. **7** (1), (2006), Article 30, 9 pp.
[17] B. C. YANG, *On an extension of Hilbert's integral inequality with some parameters*, Aust. J. Math. Anal. Appl. **1** (1), (2004) Article 11, 1–8.
[18] B. C. YANG, *On a general Hardy-Hilbert's inequality with a best value*, Chinese Annals of Math. **21A** (4), (2000), 401–408.