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ZYGMUND–TYPE INTEGRAL INEQUALITIES FOR POLYNOMIALS NOT VANISHING IN A DISC

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Abstract. The study of inequalities in various norms for polynomials and their derivatives in the plane is fundamental to geometric function theory. This paper focuses on Zygmund-type norm estimates for polynomials that do not vanish in a positive-radius disc. We establish integral norm estimates for the growth of higher-order derivatives of a polynomial in the plane, including extensions of several important inequalities of approximation theory and related inequalities established by Jain [*Turk. J. Math.* **31** (2007), 89–94].

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Paper MIA-28-17

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