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POSITIVE DEFINITE MATRIX–VALUED KERNELS AND THEIR SCALAR VALUED PROJECTIONS: COUNTEREXAMPLES

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Abstract. In this paper we show that the strictly positive definite matrix valued isotropic kernels in the circle and the dot product kernels in Euclidean spaces are not well behaved with respect to its scalar valued projections. We generalize the counterexamples that we obtained to an abstract setting by using the concepts of unitarily invariant kernels and adjointly invariant kernels, provided the existence of an aperiodic invariant function.

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