Mathematical
Inequalities
& Applications
Volume 28, Number 2 (2025), 381–401
doi:10.7153/mia-2025-28-26

ON SOME INEQUALITIES FOR THE h–FOURIER COSINE–LAPLACE DISCRETE GENERALIZED CONVOLUTION AND APPLICATIONS

HOANG TUNG* AND NGUYEN XUAN THAO

Abstract. In this article, we study some inequalities for the h-Fourier cosine-Laplace discrete generalized convolution on the time scale $\mathbb{T}_h^0$ and establish some norm estimations for this discrete generalized convolution on some function spaces. We present some sufficient conditions for the existence of the h-Fourier cosine-Laplace discrete generalized convolution. A Young-type inequality, a Saitoh-type inequality and a reverse Saitoh-type inequality for this discrete generalized convolution are obtained. As applications, we apply some of these inequalities to estimate the solutions of a class of equations of the h-Fourier cosine-Laplace discrete generalized convolution type.

Mathematics subject classification (2020): 39A12, 44A35, 45E10, 47A30.
Keywords and phrases: h-Laplace transform, h-Fourier cosine transform, generalized convolution inequality, Young-type inequality, Saitoh-type inequality.

REFERENCES

[1] R. A. ADAMS AND JOHN J. F. FOURNIER, *Sobolev Spaces*, 2nd Edition, Pure and Applied Mathematics series, Vol. **140**, Elsevier Ltd/Academic Press, 2003.
[2] M. BOHNER AND G. SH. GUSEINOV, *The h-Laplace and q-Laplace transforms*, J. Math. Anal. Appl., **365**, 1 (2010), 75–92.
[3] P. DYKE, *An Introduction to Laplace Transforms and Fourier Series*, 2nd Edition, Springer Undergraduate Mathematics Series, Springer/Springer-Verlag, 2014.
[4] A. A. EL-DEEB, H. A. ELSENNARY AND WING-SUM CHEUNG, *Some reverse Hölder inequalities with Specht's ratio on time scales*, J. Nonlinear Sci. Appl., **11**, (2018), 444–455.
[5] J. I. FUJII, S. IZUMINO AND Y. SEO, *Determinant for positive operators and Specht's theorem*, Sci. Math., **1**, 3 (1998), 307–310.
[6] S. HILGER, *An Application of Calculus on Measure Chains to Fourier Theory and Heisenberg's Uncertainty Principle*, J. Difference Equ. Appl., **8**, 10 (2002), 897–936.
[7] N. T. H. PHUONG, *The Fourier generalized convolutions on time scales $h\mathbb{N}^0$ and their applications*, Annales Univ. Sci. Budapest., Sect. Computatorica, **54** (2023), 265–280.
[8] S. SAITOH, *Weighted L_p-Norm Inequalities in Convolutions*, Survey on Classical Inequalities, by T. M. Rassias (editor), Mathematics and Its Applications, Vol. **517**, Springer, Dordrecht, (2000), 225–234.
[9] S. SAITOH, V. K. TUAN AND M. YAMAMOTO, *Reverse Weighted L_p-Norm Inequalities in Convolutions*, J. of Inequalities in Pure and Appl. Math., **1**, 1 (2000) Article 7.
[10] W. SPECHT, *Zur Theorie der elementaren Mittel*, Math. Z., **74** (1960), 91–98.
[11] E. M. STEIN AND G. WEISS, *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton University Press, Princeton, New Jersey, 1971.
[12] N. X. THAO AND L. X. HUY, *Fourier cosine-Laplace generalized convolution inequalities and applications*, Math. Inequal. Appl., **22**, 1 (2019), 181–195.
[13] N. X. THAO AND N. T. H. PHUONG, *Fourier Cosine Transform on Time Scale*, The Second Vietnam International Applied Mathematics Conference – VIAMC 2017, Ho Chi Minh City, Vietnam, 2017 Dec 15–18, 207–215, Information and Communications Publishing House, Vietnam.

Paper MIA-28-26

[14] N. X. THAO AND H. TUNG, *The generalized convolution for h-Laplace transform on time scale*, HNUE J. of Sci., Nat. Sci., **64**, 10 (2019), 17–35.
[15] N. X. THAO AND H. TUNG, *The h-Fourier cosine-Laplace generalized convolution with a weight function*, Annales Univ. Sci. Budapest., Sect. Computatorica, **54** (2023), 321–340.
[16] M. TOMINAGA, *Specht's ratio in the Young inequality*, Sci. Math. Japonicae, **55**, 3 (2002), 583–588.
[17] H. TUNG, N. X. THAO AND V. K. TUAN, *The h-Fourier sine-Laplace discrete generalized convolution on time scale*, Integral Transforms and Spec. Funct., **34**, 6 (2023), 444–456.