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NONCOMMUTATIVE DONOHO–ELAD–GRIBONVAL–NIELSEN–FUCHS SPARSITY THEOREM

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Abstract. Breakthrough Sparsity Theorem, derived independently by Donoho and Elad *[Proc. Natl. Acad. Sci. USA, 2003]*, Gribonval and Nielsen *[IEEE Trans. Inform. Theory, 2003]* and Fuchs *[IEEE Trans. Inform. Theory, 2004]* says that unique sparse solution to NP-Hard ℓ_0-minimization problem can be obtained using unique solution of P-Type ℓ_1-minimization problem. In this paper, we derive noncommutative version of their result using frames for Hilbert C*-modules.

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