

## ESTIMATES FOR MAXIMAL FOURIER MULTIPLIER OPERATORS ON $\mathbb{R}^2$ VIA SQUARE FUNCTIONS

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*Abstract.* We consider certain Littlewood-Paley square functions on  $\mathbb{R}^2$  and prove sharp estimates for them, from which we can deduce  $L^p$  boundedness of maximal functions defined by Fourier multipliers of Bochner-Riesz type on  $\mathbb{R}^2$ . This is a generalization of a result due to A. Carbery 1983.

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