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NEW BOUNDS FOR THE IDENTRIC AND LOGARITHMIC MEANS

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Abstract. We assume that the numbers x and y are positive and unequal. Let $H(x,y)$, $G(x,y)$, $L(x,y)$, $I(x,y)$, and $A(x,y)$ be the harmonic, geometric, logarithmic, identric, and arithmetic means of x and y, respectively. In this paper we present new bounds for the identric and logarithmic means. For example, we prove that the inequality $\frac{5}{6}A^p+\frac{1}{6}H^p<I^p$ holds for $0<p\leqslant 12/25$, and the reverse inequality holds for $p\in(-\infty,0)\cup[1,\infty)$. We prove $L^p<\frac{2}{3}G^p+\frac{1}{3}A^p$ for $p\in(-\infty,0)\cup[4/5,\infty)$.

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