

## CARLSON TYPE INEQUALITIES AND OPTIMAL RECOVERY OF DIFFERENTIAL OPERATORS

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*Abstract.* The paper considers a multidimensional problem of optimal recovery of an operator whose action is represented by multiplying the original function by a weight function of a special type, based on inaccurately specified information about the values of operators of a similar type. An exact inequality for the norms of such operators is obtained. The problem under consideration is a generalization of the problem of optimal recovery of a derivative based on other inaccurately specified derivatives in the space  $\mathbb{R}^d$  and the problem of an exact inequality, which is an analogue of the Hardy–Littlewood–Polya inequality.

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