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ON THE GENERALIZED n–TH JORDAN–VON NEUMANN CONSTANT AND FIXED POINT PROPERTY

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Abstract. In this paper, we introduce a new geometric constant $C_{NJ}^{(n)}(a,X)$ of a Banach space X, which is closely related to the n-th Jordan-Von Neumann constant and analyze some properties of the constant. Subsequently, we present a relationship between the weakly convergent sequence coefficient (WCS) and this new constant. Our main results of the paper improve some known results in the recent literature.

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