Mathematical
Inequalities
& Applications
Volume 29, Number 3 (2026), 243–256
doi:10.7153/mia-2026-29-14

PITT–TYPE INEQUALITY FOR $p > q$ WITH GM–CONDITION

NIYAZ TOKMAGAMBETOV

Abstract. In this paper, we examine the results of the Pitt type for the Fourier transform, specifically focusing on scenarios where the parameter p is greater than q. Here, we show that for the Pitt inequality the 'if and only if' condition established by Saucedo and Tikhonov (*The Fourier transform is an extremizer of a class of bounded operators, arXiv.2501.17505*) can be extended under restriction to the class of GM functions.

Mathematics subject classification (2020): 42B10, 26D10, 42A38.
Keywords and phrases: Pitt's inequality, bounded variation, general monotone function.

REFERENCES

[1] J. J. BENEDETTO, H. P. HEINIG, R. JOHNSON, *Weighted Hardy spaces and the Laplace transform. II*, Math. Nachr. **132** (1987), 29–55.
[2] J. J. BENEDETTO AND H. P. HEINIG, *Weighted Fourier inequalities: new proofs and generalizations*, J. Fourier Anal. Appl. **9** (2003), 1–37.
[3] A. DEBERNARDI, *The Boas problem on Hankel transforms*, J. Fourier Anal. Appl. **25**, no. 6, 3310–3341 (2019).
[4] L. DE CARLI, D. GORBACHEV AND S. TIKHONOV, *Pitt and Boas inequalities for Fourier and Hankel transforms*, J. Math. Anal. Appl. **408** (2013), no. 2, 762–774.
[5] L. DE CARLI, D. GORBACHEV AND S. TIKHONOV, *Pitt inequalities and restriction theorems for the Fourier transform*, Rev. Mat. Iberoam. **33**, no. 3 (2017), 789–808.
[6] D. GORBACHEV, E. LIFLYAND AND S. TIKHONOV, *Weighted Fourier Inequalities: Boas conjecture in* $\mathbb{R}^n$, J. d'Analyse Math., vol. 114 (2011), 99–120.
[7] H. P. HEINIG, *Weighted norm inequalities for classes of operators*, Indiana Univ. Math. J. **33** (1984), no. 4, 573–582.
[8] H. P. HEINIG, *Estimates for operators in mixed weighted* L^p *-spaces*, Trans. Amer. Math. Soc. **287** (1985), no. 2, 483–493.
[9] W. B. JURKAT AND G. SAMPSON, *On maximal rearrangement inequalities for the Fourier transform*, Trans. Amer. Math. Soc. **282** (1984), no. 2, 625–643.
[10] W. B. JURKAT AND G. SAMPSON, *On rearrangement and weight inequalities for the Fourier transform*, Indiana Univ. Math. J. **33** (1984), 257–270.
[11] A. KUFNER, L.-E. PERSSON, *Weighted inequalities of Hardy type*, Singapore: World Scientific (ISBN 981-238-195-3/hbk), xviii, 357 p. (2003).
[12] E. LIFLYAND AND S. TIKHONOV, *Extended solution of Boas' conjecture on Fourier transform*, C. R. Math. Acad. Sci. Paris **346** (2008), 1137–1142.
[13] E. LIFLYAND AND S. TIKHONOV, *A concept of general monotonicity and applications*, Math. Nachr. **284** (2011), no. 8–9, 1083–1098.
[14] B. MUCKENHOUPT, *A note on two weight function conditions for a Fourier transform norm inequality*, Proc. Am. Math. Soc. **88** (1983), no. 1, 97–100.
[15] B. MUCKENHOUPT, *Weighted norm inequalities for the Fourier transform*, Trans. Amer. Math. Soc. **276** (1983), no. 2, 729–742.
[16] E. D. NURSULTANOV, *Net spaces and inequalities of Hardy-Littlewood type*, (English. Russian original) Sb. Math. **189**, no. 3, 399–419 (1998); translation from Mat. Sb. **189**, no. 3, 83–102 (1998).
[17] J. RASTEGARI, G. SINNAMON, *Weighted Fourier inequalities via rearrangements*, J. Fourier Anal. Appl. **24** (2018), 1225–1248.

Paper MIA-29-14

[18] M. SAUCEDO, S. TIKHONOV, *The Fourier transform is an extremizer of a class of bounded operators*, preprint: https://doi.org/10.48550/arXiv.2501.17505.
[19] N. TOKMAGAMBETOV, *Pitt-type inequalities for general monotone functions*, preprint: http://hdl.handle.net/2072/483459.