

ON THE ZEROS OF A QUATERNIONIC POLYNOMIAL WITH RESTRICTED COEFFICIENTS

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Abstract. In recent times, several mathematicians are working on the estimation of zeros of polynomials with quaternionic coefficients using diverse methods. In this paper, we focus on deriving the upper bounds for the location of zeros and establish zero-free regions for specific regular functions of a quaternionic variable with restricted coefficients. To achieve our results, we utilize the Schwarz's lemma and the zero sets of a regular product established in the recently developed theory of regular functions and polynomials of a quaternionic variable. Additionally, our findings lead to generalisations of several well known results in the existing quaternionic literature.

Mathematics subject classification (2020): Primary 30E10; Secondary 30G35, 16K20.

Keywords and phrases: Quaternionic polynomial, Eneström-Kakeya theorem, Schwarz's lemma, zero-sets of a regular product.

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