

ORDER BOUNDEDNESS, ESSENTIAL NORM AND COMPACTNESS OF GENERALIZED STEVIĆ-SHARMA OPERATORS ON DERIVATIVE HARDY SPACES

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Abstract. We investigate the order boundedness of generalized Stević-Sharma operators on derivative Hardy spaces. We also establish characterizations for the boundedness, essential norm and compactness of these operators mapping from derivative Hardy spaces to Bloch-type spaces.

Mathematics subject classification (2020): 47B38, 42B30, 30H30.

Keywords and phrases: Stević-Sharma operator, derivative Hardy space, order bounded, essential norm.

REFERENCES

- [1] E. ABBASI AND M. HASSANLOU, *Generalized Stević-Sharma Type Operators on Spaces of Fractional Cauchy Transforms*, *Mediterr. J. Math.* **21**, (2024), 40.
- [2] E. ABBASI, Y. LIU AND M. HASSANLOU, *Generalized Stević-Sharma type operators from Hardy spaces into n th weighted type spaces*, *Turkish J. Math.* **45**, 4 (2021), 1543–1554.
- [3] M. D. CONTRERAS AND A. G. HERNÁNDEZ-DÍAZ, *Weighted composition operators on spaces with derivative in a Hardy space*, *J. Operator Theory* **52**, (2004), 173–184.
- [4] C. C. COWEN AND B. D. MACCLUER, *Composition operators on spaces of analytic functions*, *Studies in Advanced Mathematics*, CRC Press, Boca Raton, 1995.
- [5] J. DU, S. LI AND Z. LIU, *Stević-Sharma-type operators between Bergman spaces induced by doubling weights*, *Math. Meth. Appl. Sci.* **47**, (2024), 10006–10023.
- [6] P. DUREN, *Theory of H^p Spaces*, Academic Press, New York, 1970.
- [7] Y. GAO, S. KUMAR AND Z. ZHOU, *Order bounded weighted composition operators mapping into the Dirichlet type spaces*, *Chin. Ann. Math.* **37B**, 4 (2016), 585–594.
- [8] Z. GUO, *Generalized Stević-Sharma operators from the minimal Möbius invariant space into Bloch-type spaces*, *Demonstr. Math.* **56**, 1 (2023), 20220245.
- [9] Z. GUO AND J. MU, *Generalized Stević-Sharma type operators from derivative Hardy spaces into Zygmund-type spaces*, *AIMS Math.* **8**, 2 (2023), 3920–3939.
- [10] Z. GUO AND Y. SHU, *On Stević-Sharma operators from Hardy spaces to Stević weighted spaces*, *Math. Inequal. Appl.* **23**, 1 (2020), 217–229.
- [11] Z. GUO AND X. ZHAO, *On a Stević-Sharma type operator from $Q_k(p, q)$ spaces to Bloch-type spaces*, *Oper. Matrices* **16**, 2 (2022), 563–580.
- [12] R. A. HIBSCHWEILER, *Order bounded weighted composition operators*, *Contemp. Math.* **454**, (2008), 93–105.
- [13] R. A. HIBSCHWEILER AND N. PORTNOY, *Composition followed by differentiation between Bergman and Hardy spaces*, *Rocky Mountain J. Math.* **35**, 3 (2005), 843–855.
- [14] N. HU, *Weighted composition operators from derivative Hardy spaces into n -th weighted-type spaces*, *J. Math.* **2021**, (2021), 4398397.
- [15] H. HUNZIKER AND H. JARCHOW, *Composition operators which improve integrability*, *Math. Nachr.* **152**, (1991), 83–99.
- [16] Z. J. JIANG, *Order bounded Stević-Sharma operators between weighted Dirichlet spaces*, *Oper. Matrices* **19**, 3 (2025), 295–309.

- [17] S. LI AND S. STEVIĆ, *Composition followed by differentiation from mixed-norm spaces to α -Bloch spaces*, Sb. Math. **199**, 12 (2008), 1847–1857.
- [18] S. LI AND S. STEVIĆ, *Products of composition and differentiation operators from Zygmund spaces to Bloch spaces and Bers spaces*, Appl. Math. Comput. **217**, 7 (2010), 3144–3154.
- [19] Q. LIN, J. LIU AND Y. WU, *Order boundedness of weighted composition operators on weighted Dirichlet spaces and derivative Hardy spaces*, Bull. Belg. Math. Soc. Simon Stevin **27**, 4 (2020), 627–637.
- [20] M. LINDSTRÖM, D. NORRBO AND S. STEVIĆ, *On compactness of operators from Banach spaces of holomorphic functions to Banach spaces*, J. Math. Inequal. **18**, 3 (2024), 1153–1158.
- [21] Z. LIU, *Order boundedness and essential norm of generalized weighted composition operators on Bergman spaces with doubling weights*, Oper. Matrices **17**, 4 (2023), 995–1009.
- [22] C. LO AND A. W. LOH, *Stević-Sharma Type Operators on Bergman Spaces Induced by Doubling Weights*, Bull. Malays. Math. Sci. Soc. **48**, 2 (2025), 34.
- [23] B. D. MACCLUER, *Composition operators on $\mathcal{S}^p$* , Houston J. Math. **13**, 2 (1987), 245–254.
- [24] K. MADIGAN AND A. MATHESON, *Compact composition operators on the Bloch space*, Trans. Amer. Math. Soc. **347**, 7 (1995), 2679–2687.
- [25] S. OHNO, *Products of composition and differentiation between Hardy spaces*, Bull. Austral. Math. Soc. **73**, 2 (2006), 235–243.
- [26] R. C. ROAN, *Composition operators on the space of functions with H^p -derivative*, Houston J. Math. **4**, (1978), 423–438.
- [27] A. K. SHARMA, *Products of composition multiplication and differentiation between Bergman and Bloch type spaces*, Turk. J. Math. **35**, 2 (2011), 275–291.
- [28] A. K. SHARMA, *On order bounded weighted composition operators between Dirichlet spaces*, Positivity **21**, 3 (2017), 1213–1221.
- [29] S. STEVIĆ, *Composition operators from the weighted Bergman space to the n th weighted spaces on the unit disc*, Discrete Dyn. Nat. Soc. Art. **2009**, (2009), 742019.
- [30] S. STEVIĆ, *Norm and essential norm of composition followed by differentiation from α -Bloch spaces to H^∞_μ* , Appl. Math. Comput. **207**, 1 (2009), 225–229.
- [31] S. STEVIĆ, *Weighted differentiation composition operators from mixed-norm spaces to weighted-type spaces*, Appl. Math. Comput. **211**, 1 (2009), 222–233.
- [32] S. STEVIĆ, *Composition followed by differentiation from H^∞ and the Bloch space to n -th weighted-type spaces on the unit disk*, Appl. Math. Comput. **216**, (2010), 3450–3458.
- [33] S. STEVIĆ, *Composition operators from the Hardy space to the n th weighted-type space on the unit disk and the half-plane*, Appl. Math. Comput. **215**, (2010), 3950–3955.
- [34] S. STEVIĆ, *Weighted differentiation composition operators from the mixed-norm space to the n th weighted-type space on the unit disk*, Abstr. Appl. Anal. **2010**, (2010), 246287.
- [35] S. STEVIĆ, *Weighted iterated radial composition operators between some spaces of holomorphic functions on the unit ball*, Abstr. Appl. Anal. **2010**, (2010), 801264.
- [36] S. STEVIĆ, *On a new product-type operator on the unit ball*, J. Math. Inequal. **16**, 4 (2022), 1675–1692.
- [37] S. STEVIĆ, C. S. HUANG AND Z. J. JIANG, *Sum of some product-type operators from Hardy spaces to weighted-type spaces on the unit ball*, Math. Methods Appl. Sci. **45**, 17 (2022), 11581–11600.
- [38] S. STEVIĆ, A. K. SHARMA AND A. BHAT, *Essential norm of products of multiplication composition and differentiation operators on weighted Bergman spaces*, Appl. Math. Comput. **218**, 6 (2011), 2386–2397.
- [39] S. STEVIĆ, A. K. SHARMA AND A. BHAT, *Products of multiplication composition and differentiation operators on weighted Bergman space*, Appl. Math. Comput. **217**, 20 (2011), 8115–8125.
- [40] S. STEVIĆ, A. K. SHARMA AND R. KRISHAN, *Boundedness and compactness of a new product-type operator from a general space to Bloch-type spaces*, J. Inequal. Appl. **219**, (2016), 32 pp.
- [41] S. UEKI, *Order bounded weighted composition operators mapping into the Bergman Space*, Complex Anal. Oper. Theory **6**, 3 (2012), 549–560.
- [42] E. WOLF, *Order bounded of weighted composition operators*, J. Aust. Math. Soc. **93**, 3 (2012), 333–343.
- [43] K. ZHU, *Bloch type spaces of analytic functions*, Rocky Mountain J. Math. **23**, 3 (1993), 1143–1177.
- [44] X. ZHU, *Products of differentiation, composition and multiplication from Bergman type spaces to Bers type space*, Integr. Transforms Spec. Funct. **18**, 3 (2007), 223–231.

- [45] X. ZHU, *Weighted composition operators from Dirichlet-type spaces into Stević-type spaces*, Math. Inequal. Appl. **23**, 4 (2020), 1311–1323.
- [46] X. ZHU, E. ABBASI AND A. EBRAHIMI, *A class of operator-related composition operators from the Besov spaces into the Bloch space*, Bull. Iranian Math. Soc. **47**, 1 (2021), 171–184.