

## AN ELEMENTARY PROOF OF THE CONVEXITY OF $\log(\Gamma(x)\Gamma(1/x))$

BHUKYA RAVI

*Abstract.* We present a new elementary proof that the function  $\Gamma(x)\Gamma(1/x)$  is log-convex for  $x > 0$ . The proof is self-contained and avoids the technical estimates of polygamma functions found in existing literature.

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