

AN EXTENSION OF THE α -SEMI-NORM ON C^* -ALGEBRAS

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Abstract. In this paper, we introduce the $a_{(\alpha,\beta)}$ -semi-norm of elements in C^* -algebras, which generalizes the α -numerical radius $v_\alpha(x)$ and semi-norm $\|x\|_a$. We study some of the fundamental properties of the $a_{(\alpha,\beta)}$ -semi-norm and also get some results of its upper and lower bounds. Further, as applications of our results, we obtain some bounds on the α -numerical radius, which improves on some important inequalities that exist.

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