

LOGARITHMIC CONVEXITY AND INEQUALITIES FOR INTEGRALS OF GEOMETRICALLY CONVEX FUNCTIONS

BO-YAN XI, YAN WANG AND FENG QI*

Abstract. In the paper, the authors present logarithmic convexity and inequalities for integrals of geometrically convex functions. These results refine and generalize the celebrated Hermite–Hadamard’s integral inequality.

Mathematics subject classification (2020): Primary 26A51, 26D15; Secondary 41A55.

Keywords and phrases: Integral inequality of Hermite–Hadamard type, geometrically convex function, logarithmically convex function.

REFERENCES

- [1] M. AVRIEL, *r*-convex functions, Math. Programming **2** (1972), no. 1, 309–323, <https://doi.org/10.1007/BF01584551>.
- [2] S. S. DRAGOMIR, *Two mappings in connection to Hadamard’s inequalities*, J. Math. Anal. Appl. **167** (1992), no. 1, 49–56, [http://dx.doi.org/10.1016/0022-247X\(92\)90233-4](http://dx.doi.org/10.1016/0022-247X(92)90233-4).
- [3] S. S. DRAGOMIR, D. S. MILOŠVIĆ, AND J. SÁNDOR, *On some refinements of Hadamard’s inequalities and applications*, Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. **4** (1993), 3–10, <https://www.jstor.org/stable/43666351>.
- [4] D. GRONAU AND J. MATKOWSKI, *Geometrically convex solutions of certain difference equations and generalized Bohr-Mollerup type theorems*, Results Math. **26** (1994), no. 3–4, 290–297, <https://doi.org/10.1007/BF03323051>.
- [5] D. S. MITRINOVIĆ, *Analytic Inequalities*, Springer-Verlag, 1970.
- [6] C. P. NICULESCU AND L.-E. PERSSON, *Convex Functions and Their Applications: A Contemporary Approach*, third edition, CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, Springer, Cham, 2025, <https://doi.org/10.1007/978-3-031-71967-7>.
- [7] J. E. PEČARIĆ, F. PROSCHAN, AND Y. L. TONG, *Convex Functions, Partial Orderings and Statistical Applications*, Academic Press, 1991.
- [8] K.-L. TSENG, S.-R. HWANG, AND S. S. DRAGOMIR, *New Hermite–Hadamard-type inequalities for convex functions (I)*, Appl. Math. Lett. **25** (2012), no. 6, 1005–1009, <https://doi.org/10.1016/j.aml.2011.11.016>.
- [9] K.-L. TSENG, S.-R. HWANG, AND S. S. DRAGOMIR, *New Hermite–Hadamard-type inequalities for convex functions (II)*, Comput. Math. Appl. **62** (2011), no. 1, 401–418, <https://doi.org/10.1016/j.camwa.2011.05.023>.
- [10] K.-L. TSENG, G.-S. YANG, AND K.-C. HSU, *On some inequalities of Hadamard’s type and applications*, Taiwanese J. Math. **13** (2009), no. 6B, 1929–1948, <https://doi.org/10.11650/twjmath/1500405649>.
- [11] G.-S. YANG AND M.-C. HONG, *A note on Hadamard’s inequality*, Tamkang J. Math. **28** (1997), no. 1, 33–37, <https://doi.org/10.5556/j.tkjm.28.1997.4331>.