

CONVEXITY AND SOME NEW POWER MEAN INEQUALITIES IN CONTINUOUS FORMS WITH ILLUSTRATIONS/APPLICATIONS RELATED TO HEAT CONDUCTION IN COMPOSITE MATERIALS

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Abstract. The focus in this paper is on convexity (see [17]) related to continuous inequalities as described in the new and only book [19] on the subject. In particular, we complement the results in these books by proving some new continuous power mean inequalities in refined forms. In particular, some new applications related to heat conduction in composite materials are pointed out.

Mathematics subject classification (2020): Primary 26D15, 26D20; Secondary 35B27, 74A40.

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