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ASYMPTOTIC EXPANSION, CONTINUED FRACTION APPROXIMATION AND INEQUALITIES FOR RAÏSSOULI–REZGUI MEAN

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Abstract. In 2016 Raïssouli and Rezgui introduced a new bivariate mean Z given by

$$\forall s,t>0, \quad Z(s,t)=\frac{2(s-t)}{\exp(1-t/s)-\exp(1-s/t)}, \quad \text{with} \quad Z(s,s)=s,$$

which should be called *the Raïssouli-Rezgui mean*. In this paper, we present asymptotic expansion and continued fraction approximation for the Raïssouli-Rezgui mean. We establish inequalities for the Raïssouli-Rezgui mean

$$C(s,t)<Z(s,t)<N(s,t)$$

for $s,t>0$ with $s\neq t$, where $C(s,t)=\frac{2(s^2+st+t^2)}{3(s+t)}$ is the centroidal mean, and $N(s,t)=\frac{s^2+t^2}{s+t}$ is the contraharmonic mean.

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