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N–SYMMETRIC HOLOMORPHIC FUNCTIONS AND A REVISIT OF THE SERIES $\sum_{k=1}^{\infty} k^m z^k$

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Abstract. In this note we study which series of the form $\sum_{j=0}^{\infty} a_j(\sqrt[n]{z})^j$ have an analytic extension from the slitted unit disk S to the whole disk. We also give a short proof that the function

$$R_n(z) := \sum_{k\in\mathbb{Z}} \frac{1}{(\log z + 2k\pi i)^n},$$

for $n \in \mathbb{N}_2$, is the trace on the slitted plane of a rational function with a single pole of order n at 1, and that $(n-1)!R_n(z)$ coincides on S with $(-1)^n \sum_{k=1}^{\infty} k^{n-1} z^k$.

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